

# MODERN SPACE SITUATIONAL AWARENESS

*-- It Began with Piazzi, von Zach, and Gauss in 1801*

by Roger L. Mansfield, *MAA 50-Year Member*

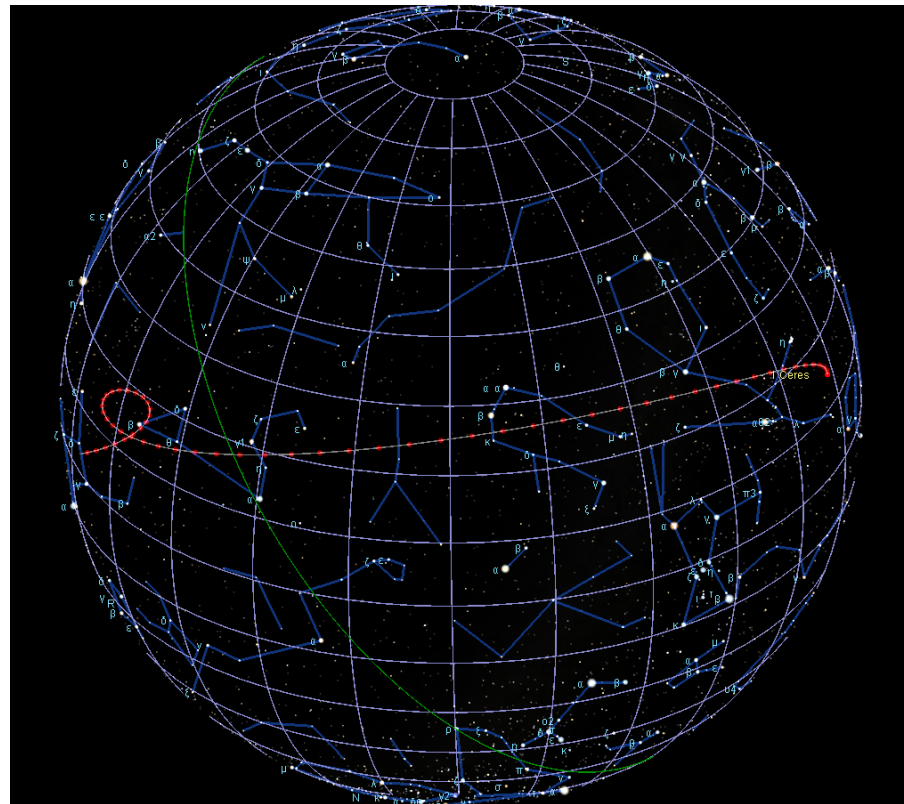
Astronomical Data Service, Colorado Springs, Colorado USA

<http://astroger.com>

*Presented at the annual meeting of the Rocky Mountain Section (RMS) of the Mathematical Association of America (MAA)*

*University of Northern Colorado  
Greeley, Colorado*

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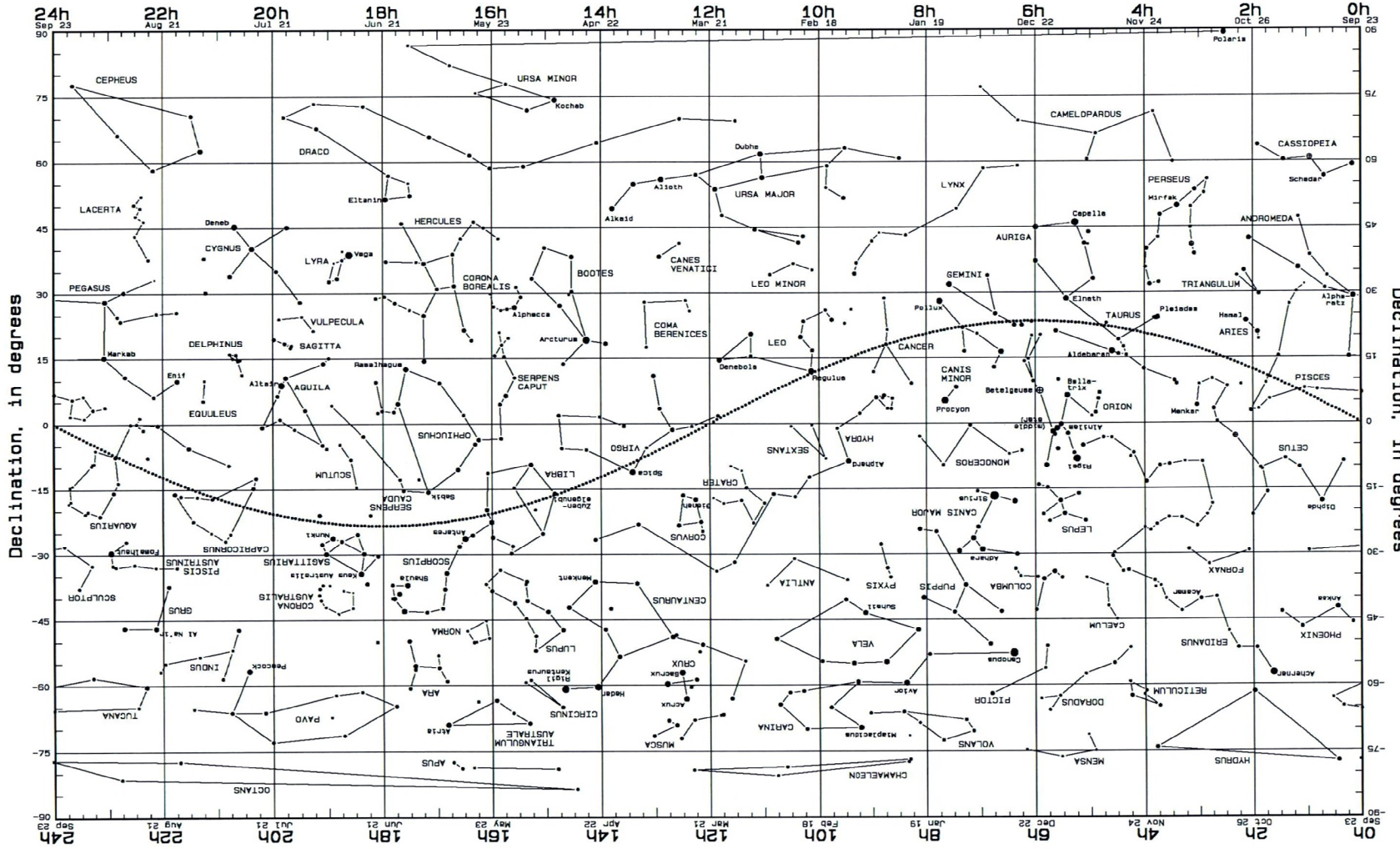
*The figure above depicts the path of the minor planet Ceres (in red) from its discovery the night of 1801 January 1 on into the year 1802*

# Space Situational Awareness - Now

- The U.S. Air Force operates a global network of *radar* and *electro-optical* sensors
- These sensors detect and track artificial Earth satellites
- The electro-optical sensors collect observations consisting of time, right ascension (RA), and declination (DEC)
- These observations go to the Joint Space Operations Center (JSpOC) at Vandenberg Air Force Base
- Using these observations, the JSpOC maintains a catalog of all deep-space objects in Earth orbit larger than about 10 cm
  - A *deep-space* object has a mean orbital motion of less than 6.4 orbital revolutions per day, whereas a *near-Earth* object has a mean orbital motion of 6.4 orbital revolutions per day or more
- Next slide is Fig. 1 - Handout Map of the Celestial Sphere

# RECTANGULAR MAP OF THE NORTH CELESTIAL HEMISPHERE (J2000)

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# RECTANGULAR MAP OF THE SOUTH CELESTIAL HEMISPHERE (J2000)

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# Space Situational Awareness - Now

- The handout map depicts the entire celestial sphere
  - RA ranges from 0 to 360 degrees (0-24 hours) and DEC ranges from -90 degrees to +90 degrees
- So every observation made by an electro-optical sensor can be plotted on this map
- The purpose of the Air Force's space catalog is to facilitate space situational awareness, i.e.,
  - what is up there in space?
  - what is it doing there?
- You can access this space catalog by going to <http://space-track.org> and creating an account see also <http://celestrak.com>

# Space Situational Awareness - 1801

- In 1801, the Italian mathematician and astronomer Giuseppe Piazzi was observing the night sky
  - using the highly-precise Palermo (Sicily) meridian circle telescope
- Piazzi's objective was to measure the right ascensions and declinations of stars, in order to compile a star catalog
- But Piazzi found a hitherto-unknown object that was moving slowly from night to night
  - Astronomers of the day, e.g., Baron Franz Xaver von Zach, thought that there might be an undiscovered major planet between Mars and Jupiter. Was this was it?
- Slide 6 depicts Piazzi's observations
  - 19 complete observations taken from the night of 1801 January 1 to 1801 February 11, as published by von Zach in his *Monatliche Correspondenz* (MC) for September 1801
- Slide 7 depicts actual path of Ceres on celestial sphere

# Space Situational Awareness - 1801

Beobachtungen des zu Palermo d. 1. Jan. 1801 von Prof. Piazzi neu entdeckten Gestirns.

1801	Mittlere Sonnen- Zeit	Gerade Aufstieg in Zeit	Gerade Auf- steigung in Gradon.	Nördl. Abweich.	Geocentri- sche Länge	Geocentr. Breite	Ost der Sonne + 20" Aberration	Logar. d. Distanz ☉ ♂
	St	St	"	"	Z	"	Z	
Jan.	1 8 43 17.8	3 27 11.25	51 47 48.8	15 37 43.5	1 23 22 58.3	3 6 42.1	9 11 1 30.9	9.9926156
	2 8 39 14.6	3 26 53.85	51 43 27.8	15 41 55.5	1 23 19 44.3	3 2 24.9	9 12 2 28.6	9.9926317
	3 8 34 53.3	3 26 38.4	51 39 36.0	15 44 31.6	1 23 16 58.6	2 53 9.9	9 13 3 26.6	9.9926324
	4 8 30 42.1	3 26 23.15	51 35 47.3	15 47 57.6	1 23 14 15.5	2 53 55.6	9 14 4 24.9	9.9926418
	10 8 6 15.8	3 25 32.1	51 28 1.5	16 10 32.0	1 23 7 59.1	2 29 0.6	9 20 10 17.5	9.9927641
	11 8 2 17.5	3 25 29.73	51 22 26.0	.....	.....	.....	.....	.....
	13 7 54 26.2	3 25 30.30	51 22 34.5	16 22 49.5	1 23 10 27.6	2 16 59.7	9 23 12 13.8	9.9928490
	14 7 50 31.7	3 25 31.72	51 22 55.8	16 27 5.7	1 23 12 1.2	2 12 56.7	9 24 14 13.5	9.9928809
	17 .....	.....	.....	16 40 13.0	.....	.....	.....	.....
	18 7 35 11.3	3 25 55.1	51 28 45.0	.....	.....	.....	.....	.....
	19 7 31 28.5	3 26 8.15	51 32 27.3	16 49 16.1	1 23 25 59.2	1 53 38.2	9 29 19 53.8	9.9930607
	21 7 24 2.7	3 26 34.27	51 38 34.1	16 58 35.9	1 23 34 21.3	1 46 6.0	10 1 20 40.3	9.9931434
	22 7 20 21.7	3 26 49.42	51 42 21.3	17 3 18.5	1 23 39 1.8	1 42 28.1	10 2 21 32.0	9.9931886
	23 7 16 43.5	3 27 6.90	51 46 43.5	17 8 5.5	1 23 44 15.7	1 38 52.1	10 3 22 22.7	9.9932348
	28 6 58 51.3	3 28 54.53	52 13 38.3	17 32 54.1	1 24 15 15.7	1 21 6.9	10 8 26 20.1	9.9935062
	30 6 51 52.9	3 29 48.14	52 27 2.1	17 43 11.0	1 24 30 9.0	1 14 16.0	10 10 27 46.2	9.9936332
	31 6 48 26.4	3 30 17.25	52 34 18.8	17 48 21.5	1 24 38 7.3	1 10 54.6	10 11 28 28.5	9.9937007
Febr.	1 6 44 59.9	3 30 47.2	52 41 48.0	17 53 36.3	1 24 46 19.3	1 7 30.9	10 12 29 9.6	9.9937703
	2 6 41 35.8	3 31 19.06	52 49 45.9	17 58 57.5	1 24 54 57.9	1 4 1.5	10 13 29 49.9	9.9938423
	5 6 31 31.5	3 33 2.70	53 15 40.5	18 15 1.0	1 25 22 43.4	0 54 23.9	10 16 31 45.5	9.9940751
	8 6 21 39.2	3 34 58.50	53 44 37.5	18 31 23.2	1 25 53 29.5	0 45 5.0	10 19 33 33.3	9.9943276
	11 6 11 58.2	3 37 6.54	54 16 38.1	18 47 58.8	1 26 26 30.0	0 36 2.9	10 22 35 13.4	9.9945823

Figure 2. Piazzi's observations of the unknown celestial object (Ceres)

The figure is an orthographic projection of the celestial sphere. 60 points of the ephemeris of Ceres are plotted at 10-day intervals. Note that the path of Ceres looped in the tail of Leo. Ceres was recovered, using the Gauss search ephemeris, as it entered the loop.

*\*Software Bisque's TheSky program was used here – see <http://bisque.com>*

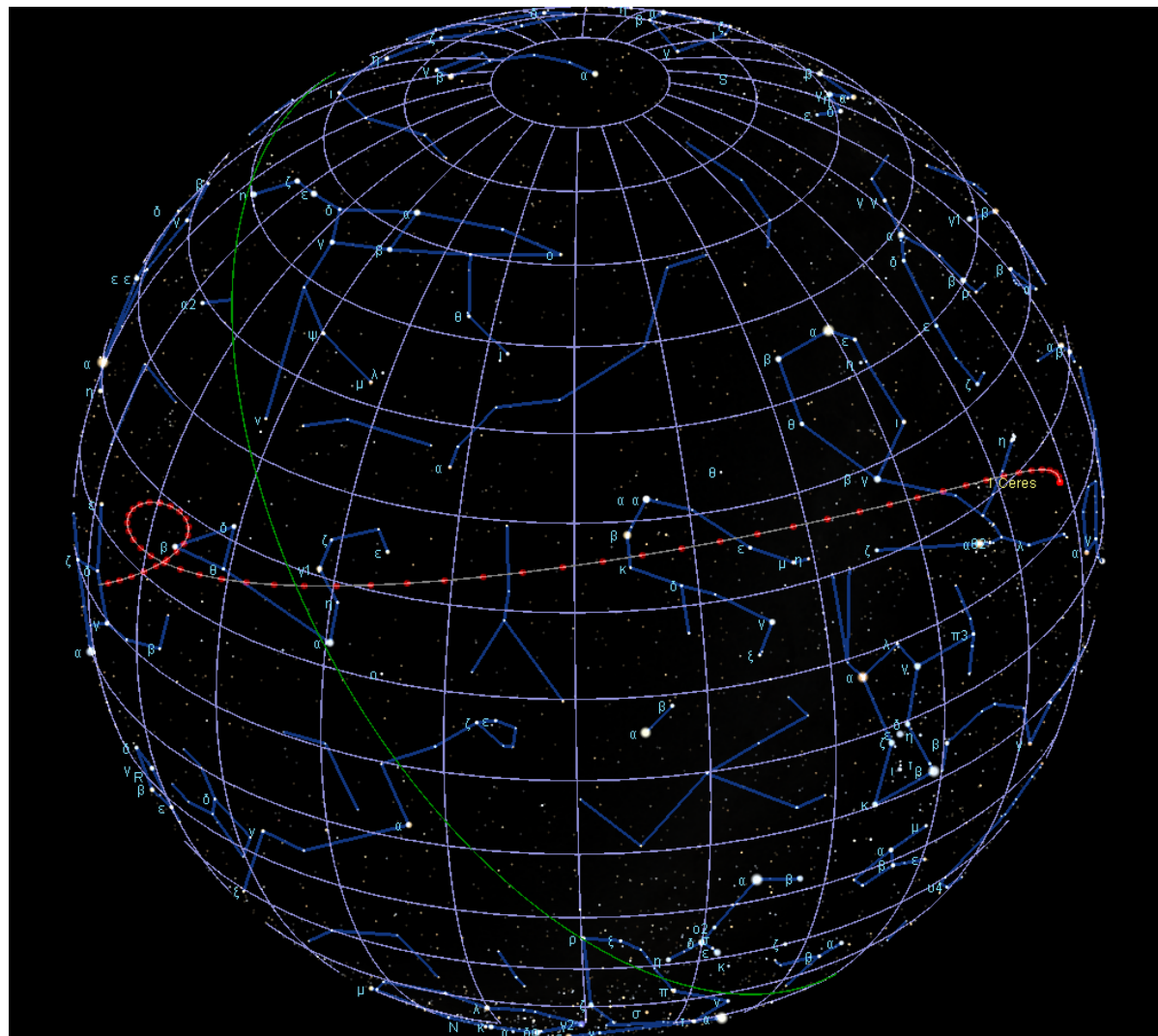


Figure 3. Path of Ceres from discovery in the constellation Taurus to recovery in the constellation Leo

# Space Situational Awareness - 1801

- The new celestial object was of great interest, but was lost from observation for almost the entire year 1801
- Carl Friedrich Gauss, mathematician, mathematical physicist, and astronomer took note of these observations and computed an orbit for the object
- Gauss's orbit put the object on a heliocentric path between the orbits of Mars and Jupiter
- Next slide shows Gauss's search ephemeris, as published by von Zach in the December 1801 issue of *Monatliche Correspondenz*



# Space Situational Awareness - 1801

Search Ephemeris of Gauss  
from *Monatliche Correspondenz*,  
Vol. 4, p. 647:

Aus diesen Elementen hat Dr. Gauss folgende  
Orter der Ceres Ferdinanda im voraus berechnet.  
Die Zeit ist mittlere für Mitternacht in Palermo.

1801	Geocentrische Länge	Geocentrische Breite nördl.	Logarith. des Abstandes von der $\odot$	Logarith. des Abstandes von der $\ominus$	Verhältniß der geoffenen Helligk.
	Z				
Nov. 25	5 20 16	9 25	0,42181	0,40468	0,6102
Dec. 1	5 22 15	9 48	0,40940	0,40472	0,6459
	7 5 24 7	10 12	0,39643	0,40479	0,6835
	13 5 25 51	10 37	0,38296	0,40488	0,7290
	19 5 27 27	11 4	0,36902	0,40499	0,7770
	25 5 28 53	11 32	0,35468	0,40512	0,8295
	31 6 0 10	12 1	0,34000	0,40528	0,8869

Z column contains "Zodiac Number" 0 through 11, to be multiplied by 30 degrees and added to degrees column

Table 4, Page 11 of my AMOS 2016 paper  
converts Gauss's geocentric ecliptic longitudes and latitudes to  
right ascensions and declinations\*

Gregorian Date	Ecliptic Longitude	Ecliptic Latitude	Right Ascension	Declination
year mo da	deg mn	deg mn	hours	degrees
1801 11 25	170 16	09 25	11.6558	12.5032
1801 12 01	172 15	09 48	11.7885	12.0665
1801 12 07	174 07	10 12	11.9141	11.6897
1801 12 13	175 51	10 37	12.0316	11.3805
1801 12 19	177 27	11 04	12.1417	11.1550
1801 12 25	178 53	11 32	12.2418	11.0116
1801 12 31	180 10	12 01	12.3331	10.9438

\*Using formulas in Chapter IV of William Marshall Smart's, *Text-Book on Spherical Astronomy*, 5<sup>th</sup> edition (Cambridge University Press, 1965), p. 40.

Figure 4. Conversion of the geocentric ecliptic longitudes and latitudes in Gauss's search ephemeris (table on left) to right ascensions and declinations (table on right)

# Space Situational Awareness - 1801

- Using Gauss's search ephemeris, von Zach observed (recovered) the new object on the night of 1801 December 31 - 1802 January 1
- Gauss became a "celebrity" throughout Europe as the result of his ingenious and extremely difficult feat of mathematical computation (with quill pen, ink, paper, and log tables!)
- Gauss had devised a method of orbit determination that was not only novel, but also of enduring interest
- See Teets and Whitehead (*Mathematics Magazine*, April 1999) for an award-winning, contemporary article that provides a historical sketch and a summary of Gauss's method:

<https://www.maa.org/programs/maa-awards/writing-awards/the-discovery-of-ceres-how-gauss-became-famous>

# Motivation and Background

- Became interested in the discovery of Ceres because of a project I was doing with Dr. Gim J. Der, whose MIT Ph.D. dissertation advisor was the great astrodynamacist Richard H. Battin (1925-2014)
- Dr. Battin was chief architect of the guidance and control hardware and software for the Apollo missions to the Moon
- Go to this link for an oral history of Dr. Battin's career:

[https://www.jsc.nasa.gov/history/oral\\_histories/BattinRH/BattinRH\\_4-18-00.htm](https://www.jsc.nasa.gov/history/oral_histories/BattinRH/BattinRH_4-18-00.htm)

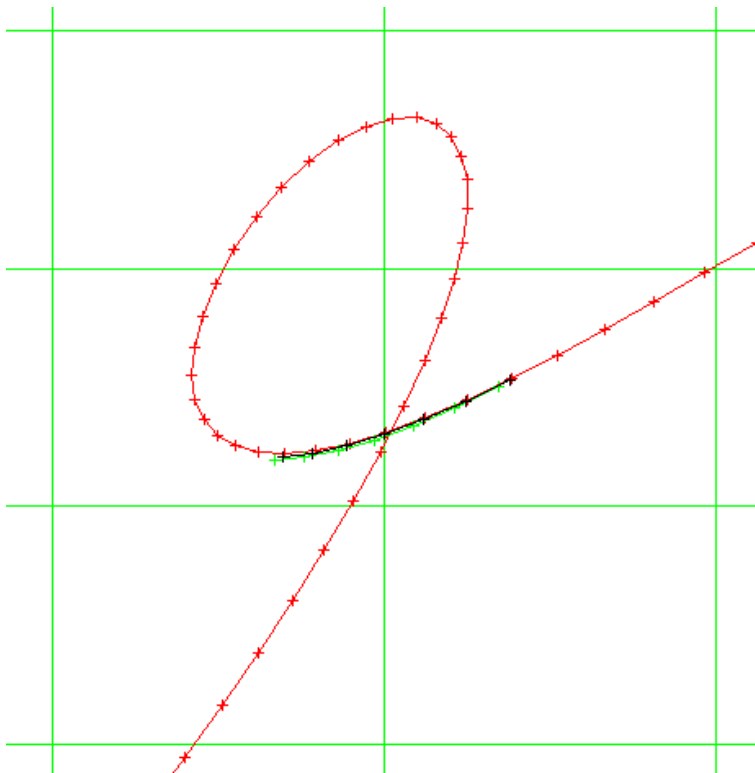
- Dr. Der and I wanted to apply some of the algorithms that we had developed for modern space situational awareness to Piazzi's observations
- We also wanted to compare our results with Gauss's results, if possible

## Difficulties, Rewards, and Results of My Research

- Was surprised and pleased to find that a public domain reprint of von Zach's *Monatliche Correspondenz* articles from 1801 had become available in the U.S. (since 2012)
- But my historical research was difficult, because von Zach's articles were
  - in German (not my native tongue, but studied in college)
  - early nineteenth-century German, at that
  - and the printed copy available to me was/is of rather poor quality
- Had not been aware that Gauss's search ephemeris had been published by von Zach
  - This was exactly what I needed to validate my own results
- Next slide depicts my results graphically

# Comparison of Contemporary vs. 1801 Results

- Red curve in Fig. 5 is the contemporary JPL Horizons-computed path of Ceres for times during 1801-1802 (best available modern ephemeris)
- Green curve is through seven points plotted from Gauss's search ephemeris
- Black curve is through the seven points computed from my own determination and differential correction of the orbit of Ceres from Piazzi's observations



- My "statistically valid" orbit for Ceres, obtained from 17 good Piazzi observations out of 19 possible, was not as good as Gauss's orbit
- But my orbit as depicted in Fig. 5, using the exact same three observations that Gauss used, was slightly better than Gauss's orbit
- I attribute the improvement to my having a better solar ephemeris in 2016 than was available to Gauss in 1801

Figure 5. Comparison of Contemporary Results with Gauss Search Ephemeris

## Summary of this MAA/RMS 2018 Presentation

- In 1801, astronomers were scanning the skies with telescopes, compiling star catalogs, and looking for new objects in orbit around the Sun
- Today, the Air Force scans the skies with telescopes -- and with radars as well -- looking for new objects in orbit around Earth
- Piazzi, von Zach, and Gauss pioneered in 1801 the methods and operational techniques of modern space situational awareness
  - because Gauss devised a new method of orbit determination still now in use,
    - for space objects in orbit around the Sun
    - for space objects detected in orbit around Earth
  - and because we use our modern, highly-precise star catalogs
    - to discriminate the unknown from the familiar, as Piazzi did,
    - and to make our observations more accurate

## References

[1] von Zach, Franz Xaver, *Monatliche Correspondenz zur Befoerderung der Erd- und Himmelskunde*, Vol. 4 (1801).

Search for the book likely only at Amazon.com, then:

[a] Piazzi's observations - September 1801 (p. 280)

[b] Gauss's search ephemeris for Ceres - December 1801 (p. 647)

[2] Mansfield, Roger L. and Gim J. Der, "Reconstruction of the 1801 Discovery Orbit of Ceres via Contemporary Angles-Only Algorithms," *Advanced Maui Optical and Space Surveillance Technologies (AMOS) Conference 2016*, Maui, Hawaii, September 20-23, 2016.

[http://astroger.com/Mansfield\\_Der\\_AMOS\\_2016\\_09\\_15\\_preprint.pdf](http://astroger.com/Mansfield_Der_AMOS_2016_09_15_preprint.pdf)

## Addenda (Time Permitting)

- How to use the handout star chart (Fig. 1, p. 3)
  - The *celestial meridian* is fixed at any observer's (e.g., your) longitude and extends from celestial north pole to celestial south pole
    - The celestial meridian can be depicted on the map as a vertical line from the upper border of the map to its lower border
  - You can simulate the celestial meridian with a pen placed on the map in vertical orientation
    - Place pen at the point on the map corresponding to tonight's date. This is the location of the celestial meridian at midnight tonight
    - Move pen one hour to the *right* for each hour *earlier* than midnight
    - Move pen one hour to the *left* for each hour *later* than midnight
    - The celestial meridian now tells you which constellations you will see above your local horizon as you look due south
  - See the third link at <http://astroger.com> for more details
- Next slide provides an overview of the Mathcad PRIME 4.0 worksheet used to convert geocentric ecliptic longitudes and latitudes of Ceres to right ascensions and declinations



## Addenda (Time Permitting)

- Conversion of geocentric ecliptic longitudes and latitudes to right ascensions and declinations
  - Let **Lambda** be geocentric ecliptic longitude and let **Beta** be geocentric ecliptic latitude
  - Let **Alpha** be right ascension and let **Delta** be declination
  - Let  $\epsilon$  be the obliquity of the ecliptic and let **JDT** be the Julian date
  - Then a Mathcad function that does the calculations is:

```

RADec(JDT, Lambda, Beta) :=
  n ← rows(Lambda)
  for i ∈ 1..n
    ε ← eps(JDTi)
    Deci ← asin(sin(Betai)·cos(ε) + cos(Betai)·sin(ε)·sin(Lambdai))
    x ← cos(Betai)·cos(Lambdai)
    y ← -sin(Betai)·sin(ε) + cos(Betai)·cos(ε)·sin(Lambdai)
    RAi ←  $\frac{\text{angle}(x, y)}{15}$ 
  augment(augment(JDT, RA·DegPerRad), Dec·DegPerRad)

```

See the Mathcad PRIME 4.0 worksheet at <http://astroger.com> for further details.

## Appendix – Two More Star Maps

- The rectangular projection on p. 3 of this presentation is not a conformal mapping of the celestial sphere to the plane
  - But it does highlight the ecliptic path of the Sun and the twelve Zodiac constellations spread out along the ecliptic
  - At the expense of greatly distorting the star regions around the north and south celestial poles
- The next two slides contain two more maps:
  - Polar equidistant projection of the north celestial hemisphere down to -45 degrees declination
  - Polar equidistant projection of the south celestial hemisphere up to +45 degrees declination
- So the two maps cover the entire celestial sphere, with 90 degrees of overlap, i.e.,
  - The first map extends 45 degrees into the southern hemisphere
  - The second map extends 45 degrees into the northern hemisphere
- And they display the north and south polar regions in the manner that we would expect to observe them



**SOUTH POLAR EQUIDISTANT PROJECTION  
OF THE CELESTIAL SPHERE, TO  
+45 DEGREES DECLINATION**

for the epoch J2000.0 A.D.

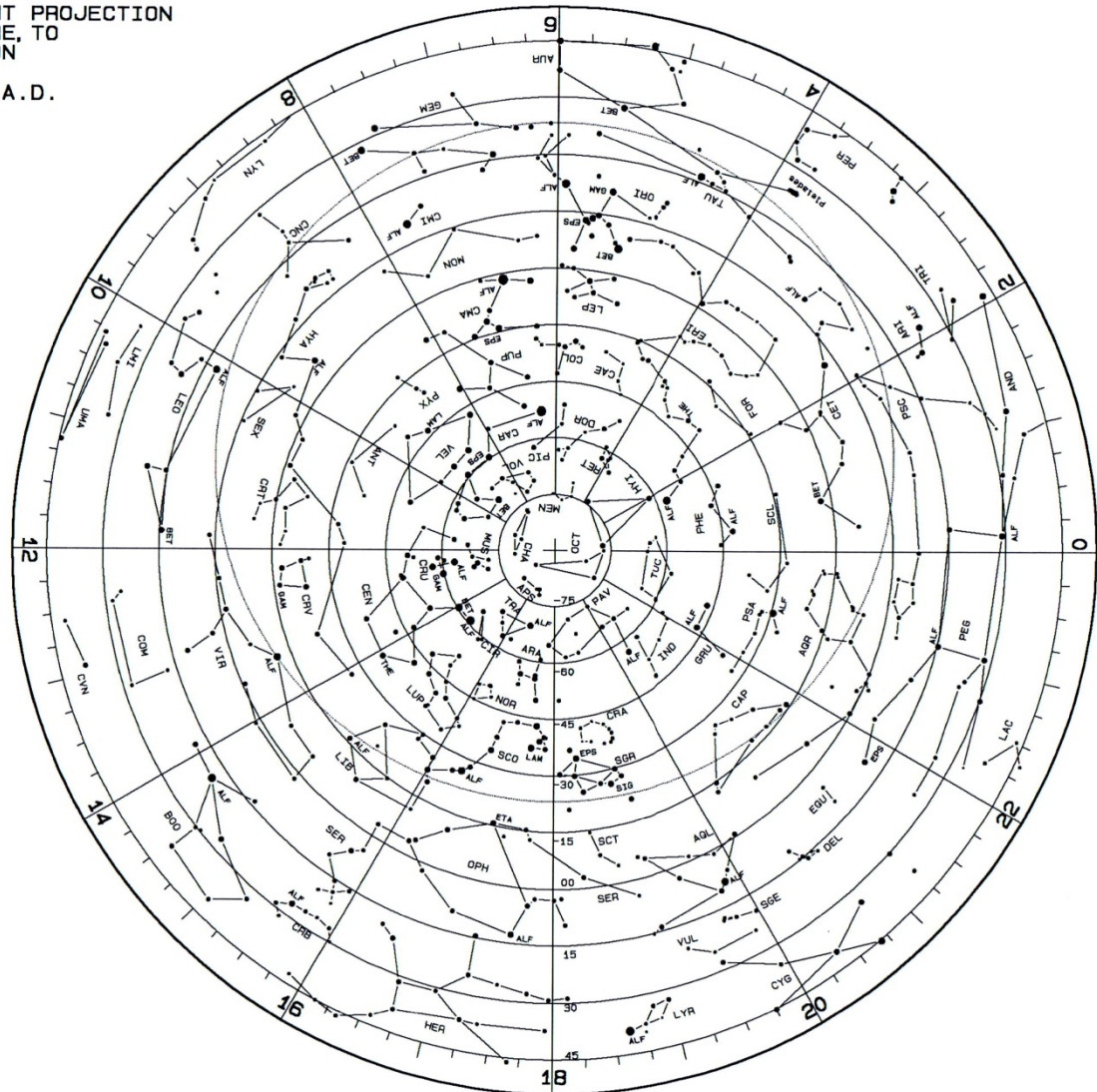
(c) 2018 by Astronomical Data Service  
Colorado Springs, Colorado U.S.A.

**NAVIGATION STARS**

Design.	Name	Magn. R.A. (h)
ALF-AND	Alpheratz	2.2 00.1
ALF-PHE	Ankaa	2.4 00.4
BET-DET	Diphia	2.2 00.7
ALF-ERI	Achernar	0.6 01.8
ALF-ARI	Hamel	2.2 02.1
THE-ERI	Acamar	3.4 03.0
ALF-DET	Menkar	2.8 03.0
ALF-TAU	Aldebaran	1.1 04.6
BET-DRI	Rigel	0.3 05.2
GAM-DRI	Bellatrix	1.7 05.4
BET-TAU	Einath	1.8 05.4
EPS-DRI	Alnilam	1.8 05.6
ALF-DRI	Betelgeuse	0-1 05.9
ALF-CAR	Canopus	-0.9 06.4
ALF-CMA	Sirius	-1.4 06.8
EPS-CMA	Adhara	1.6 07.0
ALF-CMI	Procyon	0.5 07.7
BET-GEM	Pollux	1.2 07.8
EPS-CAR	Avior	1.7 08.4
LAM-VEL	Sunbail	2.2 08.1
BET-CAR	Miaplacidus	1.8 09.2
ALF-HYA	Alphard	2.2 09.5
ALF-LEO	Regulus	1.3 10.1
BET-LEO	Denebola	2.2 11.8
GAM-DRV	Gienah	2.8 12.3
ALF-CRU	Acrux	1.6 12.4
GAM-CRU	Gacrux	1.6 12.5
ALF-VIR	Spica	1.2 13.4
BET-CEN	Hadar (Agena)	0.9 14.1
THE-CEN	Menkanti	2.3 14.1
ALF-BOO	Arcturus	0.2 14.3
ALF-CEN	Rigel Kentaurus	0.1 14.7
ALF-LIB	Zubenelgenubi	2.9 14.8
ALF-CRB	Alphecca	2.3 15.6
ALF-SCO	Antares	1.2 16.5
ALF-TRA	Atria	1.9 16.8
ETA-OPH	Sabik	2.6 17.2
LAM-SCO	Shaula	1.7 17.6
ALF-OPH	Rasalhague	2.1 17.6
EPS-SGR	Kaus Australis	2.0 18.4
ALF-LYR	Vega	0.1 18.6
SIG-SGR	Nunki	2.1 18.9
ALF-AQL	Altair	0.9 19.8
ALF-PAV	Peacock	2.1 20.4
EPS-PEG	Enif	2.5 21.7
ALF-GRU	Al Nair	2.2 22.1
ALF-PSA	Fomalhaut	1.3 23.0
ALF-PEG	Markab	2.6 23.1

**CONSTELLATIONS**

AND Andromeda  
ANT Antlia  
AFS Apus  
AGR Aqueuius  
AQL Aquila  
ARA Ara  
ARI Aries  
AUR Auriga  
BOO Bootes



**CONSTELLATIONS, CONTO.**

CAE Caelum  
CNC Cencer  
CVN Canes Venatici  
CMA Canis Major  
CMI Canis Minor  
CAP Capricornus  
CAR Carina  
CEN Centaurus  
CET Cetus  
CHA Chamaeleon  
CIR Circinus  
COL Columba  
COM Coma Berenices  
CRA Corone Australis  
CRB Corone Borealis  
CRV Corvus  
CRT Crater  
CRU Crux  
CYG Cygnus  
DEL Delphinus  
DDR Doradus  
EQU Equuleus  
ERI Eridanus  
FOR Fornax  
GEM Gemini  
GRU Grus  
HER Hercules  
HYA Hydra  
HYI Hydrus  
IND Indus  
LAC Lacerta  
LEO Leo  
LMI Leo Minor  
LEP Lepus  
LIB Libra  
LUP Lupus  
LYN Lynx  
LYR Lyra  
MEN Mensa  
MUS Musca  
MON Monoceros  
NOR Norma  
OCT Octans  
OPH Ophiuchus  
ORI Orion  
PAV Pavo  
PEG Pegasus  
PER Perseus  
PHE Phoenix  
PIC Pictor  
PSC Pisces  
PSA Piscis Austrinus  
PUP Puppis  
PYX Pyxis  
RET Reticulum  
SGE Sagitta  
SGR Sagittarius  
SCO Scorpis  
SCL Sculptor  
SCT Scutum  
SER Serpens  
SEX Sextans  
TAU Taurus  
TUC Tucana  
TRA Triangulum Australe  
TRI Triangulum  
UMA Ursa Major  
VEL Velis  
VIR Virgo  
VOL Volans  
VUL Vulpecula